Observations of elliptical galaxies and bulges show a strong correlation between the masses of their super-massive black holes (SMBH), $M_{\rm BH}$, and their stellar velocity dispersions, σ. One current idea is that this $M_{\rm BH}$–σ relation is established via SMBH feedback in gaseous protogalaxies. Models based on this idea generally consider momentum-driven or energy-driven outflows, where the gas directly traces the dark matter. Models of momentum-driven outflows in particular lead to a theoretical prediction, relating $M_{\rm BH}$ to the peak of the dark matter circular speed curve, $V_{\rm c,pk}$. To compare this prediction to the data, we consider the relation between $V_{\rm c,pk}$ and the stellar σ in local spheroids, allowing for segregation between stars and dark matter.

We solve the isotropic Jeans equation, parameterised by the stellar to dark matter mass ratio, $f(r) = M_*(r)/M_{\rm d}(r)$, for the 1-D stellar velocity dispersion, σ_r. One of our key parameters is the mass ratio inside a sphere with the stellar effective radius, $f(R_e) \equiv f_e$, which we allow to be different to the global mass ratio. We project σ_r along the line of sight, and average over a disc of radius R_e, to find the aperture dispersion, $\sigma_{\rm ap}(R_e)$, which is the velocity scale used in empirical M–σ relations, as a function of $V_{\rm c,pk}$ and f_e. Combining the theoretical M–$V_{\rm c,pk}$ relation with our Jeans modelling, we predict $M_{\rm BH}$ as a function of $\sigma_{\rm ap}(R_e)$ and compare with M–σ data. We find good agreement for $f_e \sim 6 - 16$, values that are consistent with the derived dark matter fractions from samples of local, giant ellipticals.